Department of Basic Science **Level: 1** **Examiner: Dr. Mohamed Eid** **Time allowed: 3 hours** **Answer all questions**	Pyramids higher Institute P.H.I. For Engineering And Technology معهد الأهرامات العالى للهندسة و التكنولوجيا	**Prep. Year: Final Exam** **Course: Mathematics 1** **Course Code: BAS 013 A** **Date: January 21, 2016**
The Exam consists of one page	**No. of questions: 5**	**Total Mark:** 70

Question 1

Find **y`** from the following: 18

(a) $y = 3x^3 + \cos 3x$ (b) $y = x^{-3}. \tan x$

(c) $y = \sin 2x - \tan x^2$ (d) $y = \sin x^4 + \sec^4 x$

(e) $y = \frac{2}{3} + \frac{x^2}{x^4 + \sin x}$ (f) $y = 3 + (x^5 + \frac{5}{x})^8$

Question 2

Find the limits: 8

(a) $\lim\limits_{x \to 1} \frac{1 - \sqrt{x}}{2 - x^2}$ (b) $\lim\limits_{x \to 0} \frac{x^8 - \tan^7 x}{x^7 + \sin^7 x}$ (c) $\lim\limits_{x \to 0} \frac{\sin x^3}{x^5 - 2x^3}$ (d) $\lim\limits_{x \to \infty} \frac{3 - x^5}{x + 2x^6}$

Question 3

(a)Sketch the curve of each function : $f(x) = \frac{1}{\sqrt{x^2 - 1}}$, $g(x) = \frac{x}{x^2 + 4}$ 10

(b)State and verify the mean value theorem, $f(x) = x + \frac{2}{x}$, in [1, 2]. 4

(b) Write the Maclurin's expansion of the function $f(x) = x. \cos 3x^2$. 4

Question 4

(a)State the definition of the hyperbola. 2

(b)Write the equation of circle where the points (3, 2), (0, –2) are ends of diameter. 3

(c)Show that the following circles are orthogonal and find the radical axis $x^2 + y^2 + 2x + 4y - 5 = 0$, $x^2 + y^2 - 6x + 5y + 9 = 0$. 4

(d)Write the equation of parabola with focus F(2, 1) and directrix y + 2 = 0 3

Question 5

(a) State the definition of the ellipse. 2

(b)Find the angle between the lines: $4x^2 - y^2 = 0$ and separate them. 2

(c)Find center, vertices and sketch the ellipse : 5

$9x^2 + y^2 - 36x + 6y + 36 = 0.$ (d)Find center, vertices and sketch the hyperbola : $4x^2 - y^2 + 24x + 4y + 36 = 0.$	5

Good Luck *Dr. Mohamed Eid*

Answer

Answer of Question 1

(a) $y` = 9x^2 - 3\sin 3x$

(b) $y` = x^{-3}sec^2x - 3x^{-4}\tan x$

(c) $y` = 2\cos 2x - 2x\ sec^2x^2$

(d) $y` = \cos x^4 . 4x^3 + 4sex^4x . \tan x$

(e) $y` = 0 + \frac{(x^4+\sin x).2x - x^2(4x^3+\cos x)}{(x^4+\sin x)^2}$

(f) $y` = 8\left(x^5 + \frac{5}{x}\right)^7 . (5x^4 - \frac{5}{x^2})$

--*18-Marks*

Answer of Question 2

(a) $\lim\limits_{x\to 1} \frac{1-\sqrt{x}}{2-x^2} = \frac{1-1}{2-1} = 0$

(b) $\lim\limits_{x\to 0} \frac{x^8 - \tan^7 x}{x^7 + \sin^7 x} = \frac{0}{0} = \lim\limits_{x\to 0} \frac{x - \frac{\tan^7 x}{x^7}}{1 + \frac{\sin^7 x}{x^7}} = \frac{0-1}{1+1} = -\frac{1}{2}$

(c) $\lim\limits_{x\to 0} \frac{\sin x^3}{x^5 - 2x^3} = \frac{0}{0} = \lim\limits_{x\to 0} \frac{\frac{\sin^3 x}{x^3}}{x^2 - 2} = \frac{1}{0-2} = -\frac{1}{2}$

(iv) $\lim\limits_{x\to\infty} \frac{3 - x^5}{x + 2x^6} = \frac{-\infty}{\infty + \infty} = \lim\limits_{x\to\infty} \frac{\frac{3}{x^5} - 1}{\frac{1}{x^4} + 2x} = \frac{0-1}{0+\infty} = 0$

--*8-Marks*

Answer of Question 3

(a) $f(x) = \frac{1}{\sqrt{x^2-1}}$. Domain f is R – (–1, 1).

- Asymptotic lines

The two lines $x = 1$ and $x = -1$ are vertical asymptotes.

Since $\lim_{x \to \pm\infty} \frac{1}{\sqrt{x^2 - 1}} = 0$

Then, the two lines $y = 0$ is horizontal asymptote.

No inclined asymptotes.

- Points of intersection

With x – axis: solve $\frac{1}{\sqrt{x^2-1}} \neq 0$, No points.

With y –axis: $f(0) = \frac{0}{\sqrt{0-1}} = 0$, but $x = 0$ outside the domain, No points.

- Extrema

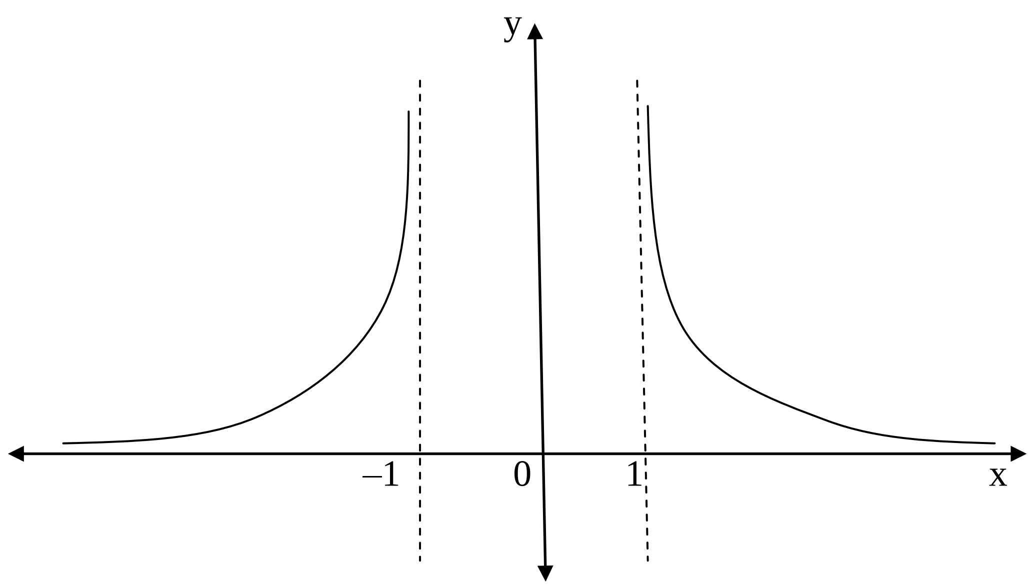
$$Since\ \ f`(x) = \frac{\sqrt{x^2-1}.0-1\frac{2x}{2\sqrt{x^2-1}}}{x^2-1} = \frac{-x}{(x^2-1)\sqrt{x^2-1}} = 0.$$ No Extrema.

Then $x = 0$, but $x = 0$ outside the domain, No Extrema.

No Inflection points.

- It is even function because $f(-x) = f(x)$.

See the figure.



$g(x) = \frac{x}{4+x^2}$. Domain f is R.

- <u>Asymptotic lines</u>

No vertical asymptotes.

Since $\lim\limits_{x \to \pm\infty} \frac{x}{1+x^2} = 0$

Then, the line y = 0 is horizontal asymptote.

No inclined asymptotes.

- <u>Points of intersection</u>

With x – axis: solve $\frac{x}{1+x^2} = 0$, we get x = 0.

With y – axis: $f(0) = \frac{0}{1+0} = 0$, then y = 0.

- Extrema

Since $f`(x)=\frac{(4+x^2).1-x.2x}{(4+x^2)^2}=\frac{4-x^2}{(1+x^2)^2}=0$. Then $4-x^2=0$

We get the point (2, 1/4) which is maximum and (–2, –1/4) which is minimum.

- Inflection points

$$Since\ \ f``(x)=\frac{(4+x^2)^2.-2x-(4-x^2).2(4+x^2).2x}{(4+x^2)^4}=\frac{2x^3-24x}{(4+x^2)^3}=0$$

Then $2x^3-24x=0$

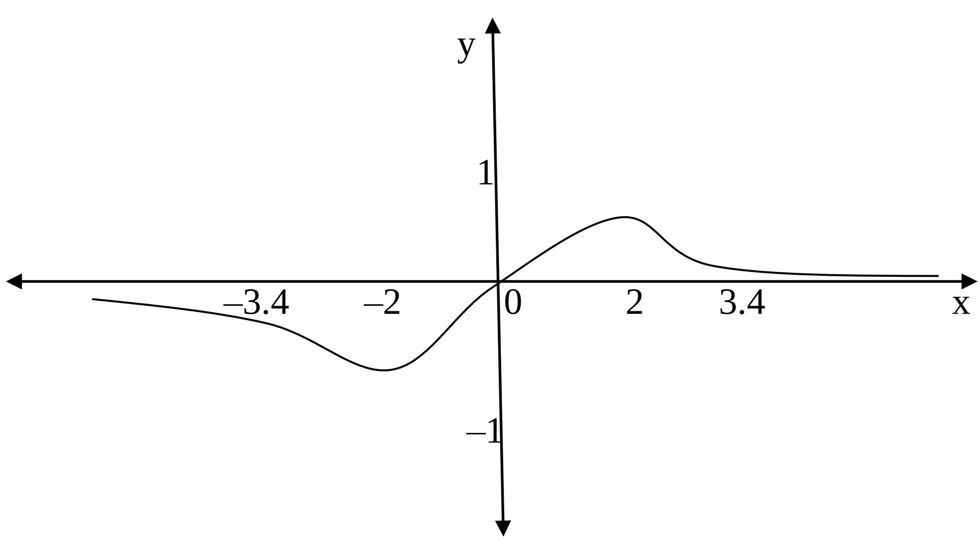
Then $x=0,2\sqrt{3}=3.4,-2\sqrt{3}=-3.4$

We get the inflection points:

$(0,0),\quad \left(2\sqrt{3},\ \frac{\sqrt{3}}{8}\right)=(3.4,\ 0.22),\quad \left(-2\sqrt{3},\ \frac{-\sqrt{3}}{8}\right)=(-3.4,\ -0.22)$

- It is odd function because $f(-x)=f(x)$.

See the figure.



--*10-Marks* (b)The mean value theorem.

We see that $f(x) = x + \frac{2}{x}$ and its derivative $f`(x) = 1 + \frac{1}{x^2}$ are continuous in the given interval [1, 2].

Then $f`(c) = 1 - \frac{2}{c^2} = \frac{f(2)-f(1)}{2-1} = \frac{3-3}{1} = 0.$

Then $c^2 = 2$ and $c = \pm\sqrt{2}.$

We see that $c = \sqrt{2}$ in [1, 2]. Then, the theorem is verified.

---*4-Marks*

(c)Since $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots.$

Then $\cos 3x^2 = 1 - 9\frac{x^4}{3!} + 81\frac{x^8}{5!} \ldots.$

Then $f(x) = x\cos 3x^2 = x - 9\frac{x^5}{3!} + 81\frac{x^9}{5!} \ldots.$

---*4-Marks*

Answer of Question 4

(a)Definition of the hyperbola.

---*2-Marks*

(b)The equation of the circle is : $(x-3)(x-0) + (y-2)(y+2) = 0$

Or $x^2 + y^2 - 3x - 4 = 0$

---*3-Marks*

(c)Since $g_1g_2 + f_1f_2 = (1)(-3) + (2)\left(\frac{5}{2}\right) = 2$ and $\frac{1}{2}(c_1 + c_2) = \frac{1}{2}(-5+9) = 2$

Then the circles are orthogonal.

The radical axis is : $8x - y - 14 = 0$.

---*3-Marks*

(d)Write the equation of parabola with focus F(2, 1) and directrix $y + 2 = 0$

The equation of parabola is given by : $\overline{PF} = \overline{PM}$

Then $\sqrt{(x-2)^2 + (y-1)^2} = \frac{y+2}{1}$

We get $x^2 - 4x + 4 + y^2 - 2y + 1 = y^2 + 4y + 4$ Or $x^2 - 4x - 6y + 1 = 0$

---*3-Marks*

Answer of Question 5

(a)Definition of the ellipse.

---*2-Marks*

(b) $\tan\theta = \pm 2\frac{\sqrt{h^2-ab}}{a+b} = \pm 2\frac{\sqrt{0+4}}{4-1} = \pm\frac{4}{3}$ real

The two lines are : $2x + y = 0$, $2x - y = 0$.

---*2-Marks*

(c)The given equation takes the form :

$$9(x^2 - 4x) + y^2 + 6y = -36$$

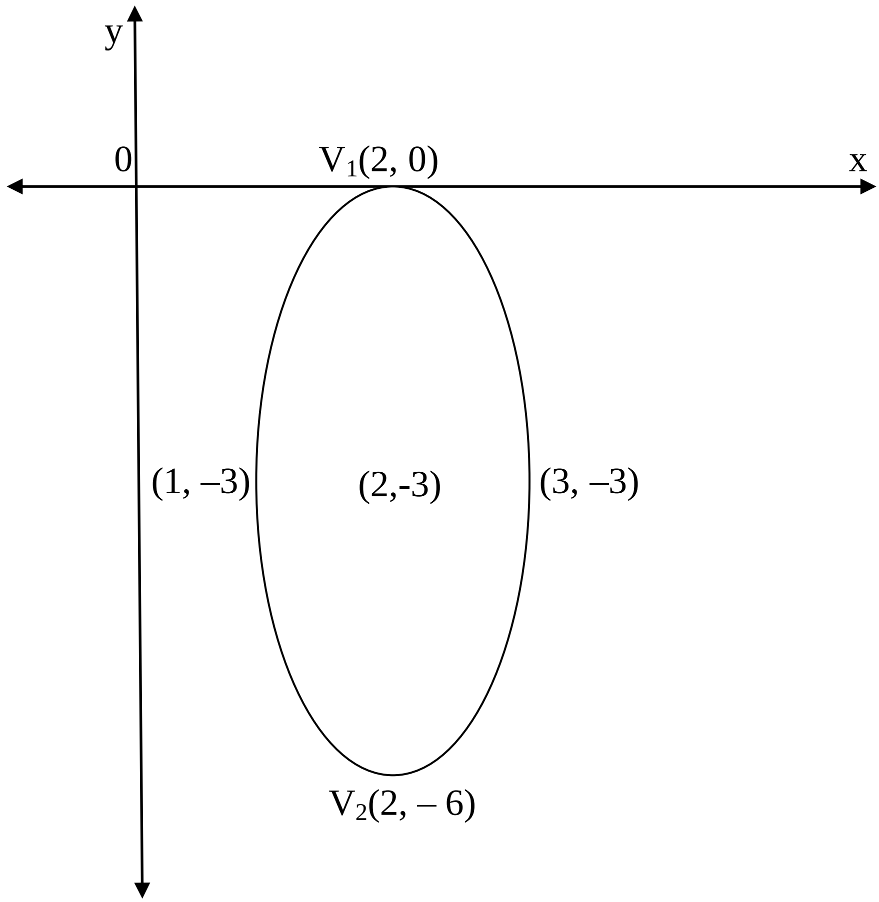
Then $9(x-2)^2 + (y+3)^2 = -36 + 36 + 9 = 9$

Then $(x-2)^2 + \frac{(y+3)^2}{9} = 1$ Vertical

We see that, center $(2, -3)$, $a = 1$, $b = 3$, $V_1 = (2, 0), V_2 = (2, -6)$

$M_1 = (1, -3), M_2 = (3, -3)$

See the figure.



--*5-Marks*

(d)The given equation takes the form :

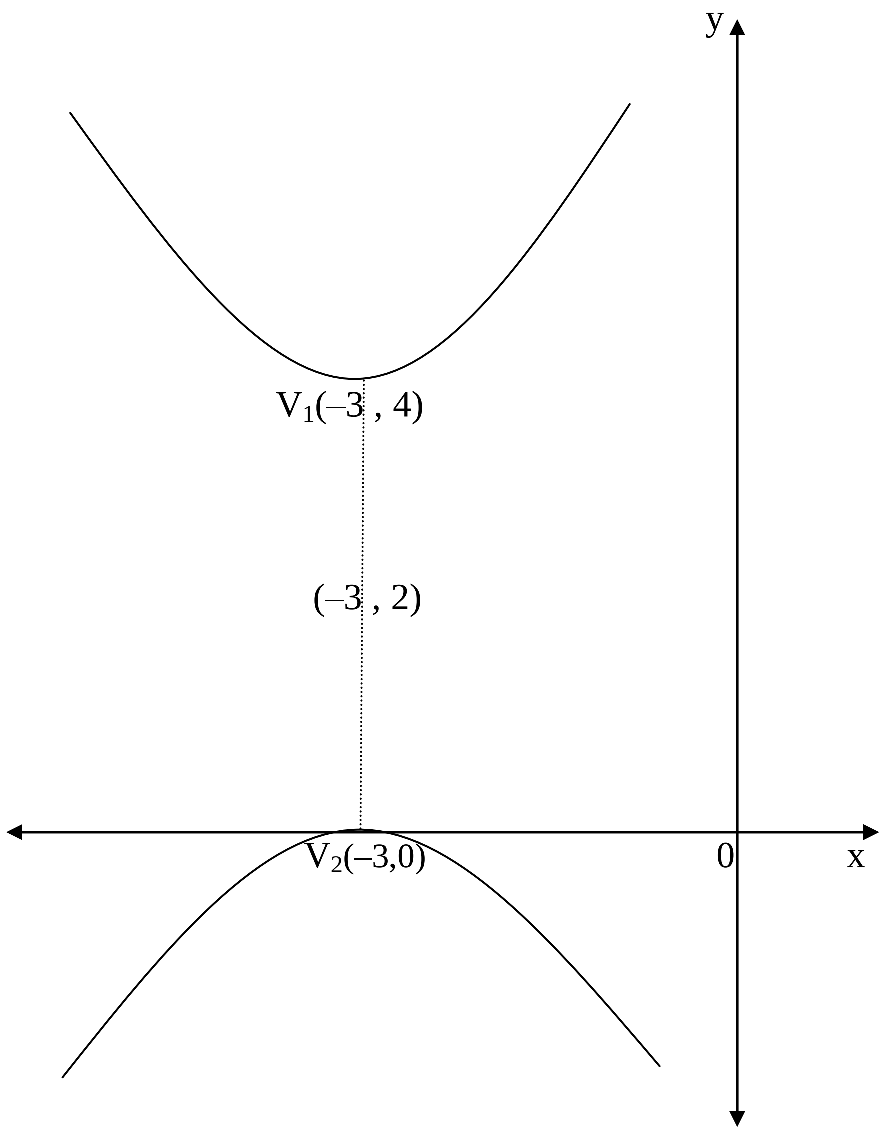
$$4(x^2+6x)-(y^2-4y)=-36$$

Then $4(x+3)^2-(y-2)^2=-36+36-4=-4$

Then $\frac{(y-2)^2}{4}-(x+3)^2=1$ <u>Vertical</u>

We see that, center (–3, 2), b = 2, $V_1=(-3,4), V_2=(-3,0)$

See the figure.



---*5-Marks*